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ONE-DIMENSIONAL ELECTROHYDRODYNAMIC FLOWS WITH VARIABLE MOBILITY COEFFICIENT. EVAPORATION AND CONDENSATION JUMPS

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V. V. Gogosov
(Moscow)
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Electrohydrodynamic flows with variable mobility coefficient and small interaction parameter are investigated. It is shown that the charge density and the electric field may considerably fluctuate in the region of variation of the stream mobility coefficient. The structure of electrohydrodynamic condensation and evaporation jumps is examined. It is shown that surface charges may appear in evaporation and condensation jumps, when the mobility coefficient varies in these. In such cases the equations derived from laws of conservation at the discontinuity are insufficient for the determination of the surface charge intensity and other governing parameters downstream of the jump. As in the case of electrohydrodynamic shock waves [1], the surface charge intensity and the related components of the electric field downstream of the discontinuity, normal to the latter, remain indeterminate, and it is necessary to supplement such equations by conditions derived by the analysis of the structure of discontinuities.

The existence of a class of electrohydrodynamic shock waves and condensation and evaporation jumps is noted. The existence of these requires that the normal component of the electric field upstream of the wave satisfies certain conditions. To determine the electrohydrodynamic parameters downstream of the shock wave requires that the electric field normal component is specified in that region. Limits within which this component can be specified are indicated.

The mobility coefficient is a complex function of thermodynamic and electrodynamic parameters and of the mixture composition. In a mixture of neutral particles and ions the mobility coefficient depends on the pressure of neutral particles and in a strong electric field, also, on the field intensity [2, 3]. Condensation and evaporation jumps can occur in channels of electrohydrodynamic generators [4]. Downstream of a condensation jump the ions stick to drops whose velocity is close to that of gas. This results in a considerable reduction of the mobility coefficient, while the evaporation of drops in the stream may increase that coefficient.

1. Let us consider one-dimensional electrohydrodynamic flows with variable mobility coefficient and assume at the beginning that the interaction parameter is small. The gasdynamic parameters, such as velocity, density, temperature, etc., are then known functions of x. On these assumptions the equations defining parameters of the electric field pattern and the charge density are

$$\frac{dE}{dx} = \frac{4\pi j}{u + bE}, \quad j = q(u + bE), \qquad j = \text{const} \tag{1.1}$$

where $u(x) > 0$, $E(x)$ and j are projections on the x-axis of velocity, electric field intensity, and current density, respectively; projections of these magnitudes on the y- and z-axes are assumed to be zero; $q(x) > 0$ is the charge density, and $b(x) > 0$ is the mobility coefficient which is assumed to be a known function of x. In what follows we consider cases in which $b = b_1 = \text{const}$ for $x \leqslant x_1$ and $b = b_2 = \text{const}$ for $x \geqslant x_2$, while in the interval $x_1 < x < x_2$ the mobility coefficient $b(x)$ either monotonically increases or decreases. Equations (1.1) are conveniently presented in the dimensionless form

$$\frac{dE^*}{d\xi} = \frac{\varepsilon}{b^*E^* + R_q u^*}, \qquad j^* = q^*(b^*E^* + R_q u^*) \tag{1.2}$$

$$\xi = \frac{x}{l}, \quad l = |x_2 - x_1|, \quad E^* = \frac{E}{E_1}, \quad q^* = \frac{q}{q_1}, \quad b^* = \frac{b}{b_1}$$

$$u^* = \frac{u}{u_1}, \quad R_q = \frac{u_1}{b_1 E_1}, \quad j^* = \frac{j}{q_1 b_1 E_1}, \quad \varepsilon = \frac{4\pi l j}{b_1 E_1^2}$$

where subscript 1 denotes parameters at $x = x_1$. The first of Eqs. (1.2) can be written as $dE^* / d\xi = Qq^*$ where $Q = 4\pi\, lq_1 / E_1$. If in this case $q_1 > 0$ and $E_1 < 0$, the derivative $dE^* / d\xi < 0$.

Let us first consider a flow, when the electric field for $\xi = \xi_1$ is negative, $E_1 < 0$ and $R_q < 0$. Let mobility $b(x)$ increase along the stretch ξ_1, ξ_2. The general pattern of behavior of integral curves defined by the first of Eqs. (1.2) is shown in Fig. 1 for $j > 0$ and in Figs. 2 and 3 for $j < 0$. The behavior of these curves for increasing mobility $b(x)$ and $j > 0$ is shown in Fig. 4. In these figures the lines denoted by I are isoclines along which derivative $dE^* / d\xi$ tends to infinity. These lines are defined by the equation

$$E^* = -R_q u^*(\xi)/b^*(\xi) \tag{1.3}$$

Above the isoclines $E^* > -R_q u^*/b^*$. The integral curves in Figs. 1, 2 and 4 relate to $|\varepsilon| \ll 1$ and the derivative $|dE^*/d\xi| \ll 1$ everywhere, except in the neighborhood of line I. The behavior of integral curves is obviously unaffected by any arbitrary changes of $|\varepsilon|$, only their slope varies. Let us assume that the velocity is constant, then $u^* = 1$. The case when the velocity is a specified function of ξ will be considered later. However, all formulas will be written in the general form to make them applicable to variable velocities, hence u^* is nowhere else assumed equal to unity.

Let us examine a few examples of flows corresponding to integral curves defined by the first of Eq. (1.2) and plotted in Fig. 1. Along segment $\xi_1 \xi_2$ the mobility coefficient increases. The tangent of the isocline angle is negative along this segment and only curves lying below isocline I, shown by solid lines, have any physical meaning. Let at a certain stream cross section $\xi = \xi_0 < \xi_1$ the electrical field $E^*(\xi_0)$ be defined by the integral curve (curve 1 in Fig. 1) along which the inequality

$$-\frac{R_q u^*(\xi_2)}{b^*(\xi_2)} < E^*(\xi_0) < -\frac{R_q u^*(\xi_1)}{b^*(\xi_1)} \tag{1.4}$$

is satisfied.

With increasing ξ downstream of this point the electrical field is weakening (since $|\varepsilon| \ll 1$ and $|b^* E^* + R_q u^*| \gg \varepsilon$) in accordance with formula (1.2). From point $\xi = \xi_1$ the mobility coefficient b^* begins to increase, while the denominator $|b^*E^* + R_q u^*|$ decreases; the integral curves approach the abscissa axis. When $|b^*E^* + R_q u^*|$ becomes of order $|\varepsilon|$ (integral curves come fairly close to the isocline I), the order of magnitude of derivative $|dE^* / d\xi|$ becomes equal unity, and the integral curves lie close to the isocline up to $\xi = \xi_2$. From thereon the coefficient of mobility remains unchanged, while E^* continues to decrease, hence the term $b^*E^* + R_q u^*$ increases in absolute value, the slope of integral curves decreases, and the order of magnitude of $|dE^*/d\xi|$ becomes equal ε. The increase of the slope of integral curves in the isocline neighborhood results in a sharp increase of the charge density. Abrupt increases of charge density are thus possible throughout a certain zone $\xi_1\xi_2$ of flow or in a part of it, when the mobility coefficient increases. Further downstream beyond point ξ_2, where mobility becomes constant, the charge density begins to decrease again, reaching the order of magnitude of the charge density in region $\xi < \xi_1$.

The physical meaning of the described phenomenon is explained in this case as follows. The electric field is negative, while the velocity and current density are positive; the ions are carried together with neutral particles by friction. The sign of the term in parentheses in Ohm's law (the second of Eqs. (1.1)) is positive, the term bE in these increases in absolute value, and the term $u + bE$ tends to zero. Since the current density j is constant, the charge density q increases, the velocity of ion motion $v_i = j / q$ diminishes, and the derivative $dE/dx = 4\pi q$ increases. With increasing ξ the negative field E may become positive. When ε is not small, a decrease of the term in parentheses also increases the change of the electric field.

If the mobility variation occurs in a narrow region, the latter may be considered to be a discontinuity. The above described behavior of integral curves provides an indication of the structure of such discontinuity. From Eqs. (1.1) for the relationship at the discontinuity we obtain

$$E_2 - E_1 = 4\pi\sigma, \quad q_1 (u_1 + b_1E_1) = q_2 (u_2 + b_2E_2) \qquad (1.5)$$

where σ is the surface charge. Obviously, $u_1 = u_2$, when $u = \text{const}$.

For the determination of the three unknowns E_2, q_2 and σ we have only two equations. The surface charge and, consequently, the electric field intensity downstream of the discontinuity remain indeterminate. The missing equation is found from the above analysis of the structure of such discontinuity.

In fact, let the width $l = |\xi_2 - \xi_1|$ of the region of mobility change decrease for constant b_1 and b_2. The slope of integral curves in this region and, consequently, the charge density increase in this region. For $l \to 0$, $\varepsilon \to 0$ and the integral curves leaving region $\xi_1 \xi_2$ have vertical tangents, the density charge downstream of the shock wave front tends to infinity, which indicates the presence of a surface charge in that region. The pattern of integral curve behavior is in this case the same as in the structure of an electrohydrodynamic shock wave with $b = \text{const}$, when a surface charge is formed within it [1].

At the limit downstream of the discontinuity front $q_2 = \infty$. From the second and third relations of Eqs. (1.1) follows that downstream of the discontinuity front

$$u_2 + b_2E_2 = 0, \qquad \sigma = -\frac{u_2}{4\pi b_2} - \frac{E_1}{4\pi} \qquad (1.6)$$

The charge density q rapidly decreases with distance from the discontinuity surface.

A discontinuity of the described kind is the simplest model of an electrohydrodynamic shock in which evaporation is taken into account. In fact, let a gas, containing liquid drops with ions collected by these in the electric field, flow into the discontinuity. The mobility coefficient in such mixture is low and the drops move at a velocity close to that of gas. If the drops evaporate downstream of the shock wave front, the mobility coefficient increases there. At the shock wave front the velocity, temperature and pressure undergo a change, and the relationships at the discontinuity are the same as in a conventional electrohydrodynamic wave [1], except that it is necessary to add to the equation of energy the term which defines the evaporation heat absorption.

For an arbitrary interaction parameter these relationships are

$$
\begin{gathered}
uv^{-1} = m = \mathrm{const}, \qquad m\{u\} + \{p\} - \frac{1}{8\pi}\{E^2\} = 0 \\
\left\{\frac{\gamma}{\gamma-1}\,pv\right\} + \frac{1}{2}\{u^2\} + \{w_0\} = 0 \\
\{E\} = 4\pi\sigma, \qquad \{q(u + bE)\} = 0 \qquad \{a\} = a_2 - a_1
\end{gathered}
\tag{1.7}
$$

where v is the specific volume, p is the pressure, $\gamma = c_p / c_v$ and $w_{01} - w_{02}$ is the heat absorbed during evaporation or released during condensation per unit of mass. These equations are written on the assumption that the mass of drops (in condensation jumps for which these equations are valid, the mass of vapor being condensed) is small in comparison with the mass of carrier gas. Hence it is possible to consider gases 1 and 2 as perfect and with the same specific heats.

If the interaction parameter is small, the last term in the second of Eqs. (1.7) can be neglected, and the system of equations determining the gasdynamic properties downstream of the discontinuity become detached from the system which defines E_2, q_2 and σ (the last two of Eqs. (1.7)). We are thus short of one equation for determining σ.

The electric field and the surface charge intensity downstream of the discontinuity front are determined by Eqs. (1.6). In fact, the formulas for which the last two of Eqs. (1.7) were derived are the same as Eqs. (1.1). The pattern of integral curve behavior in Fig. 1 which is defined by these equations, when u is a given function of x, is the same as for $u = \mathrm{const}$. This is related to the velocity decrease within the discontinuity, as happens for example in a shock wave, the term $(u + bE)$ tends to zero even more rapidly than for $u = \mathrm{const}$, when the mobility increases.

The formation of a surface charge at the mobility coefficient jump, when the mobility increases, does not necessarily always occur. The determination of its occurrence is easy, if parameters upstream of the jump and the mobility coefficient downstream of it are known. For small interaction parameter the velocity u_2 downstream of the jump is determined by the usual gasdynamic formulas. If the term $u_2 + b_2E_1$ is negative, then within the structure of such jump a surface charge must be generated (otherwise the sign of current j would be changed). Its intensity is determined by the second of Eqs. (1.6). The obtained inequality coincides with one of the inequalities (1.4), while the second of inequalities (1.4) indicates that $j > 0$.

If the velocity within the discontinuity increases, the slope of the isocline along segment $\xi_1\xi_2$ may become positive, and the integral curve pattern changes. It is then the same as in the case of condensation jumps considered below (see Fig. 4).

Let the electric field at a certain cross section $\xi = \xi_0$ be defined by the integral curve (curve 2), for which

$$
E^*(\xi_0) < -\frac{R_q u^*(\xi_2)}{b(\xi_2)}
\tag{1.8}
$$

The electric field then decreases monotonically with increasing ξ and the integral curve slope increases with increasing b^*, while the term $|b^*E^* + R_q u^*|$ remains considerably greater than ε. It will be readily seen that in this case for $\varepsilon \to 0$ such integral curves have nowhere vertical tangents, hence the charge density q is finite and at the incipient discontinuity its intensity is zero, while the electric field remains continuous

$$\sigma = 0, \quad E_1 = E_2 \tag{1.9}$$

The charge density q_2 downstream of the discontinuity front is determined by the second formula of (1.5). Formulas (1.9) are valid also for discontinuities in which u is varying.

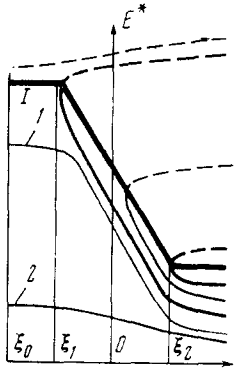

Fig. 1

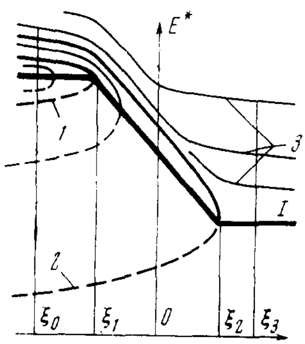

Fig. 2

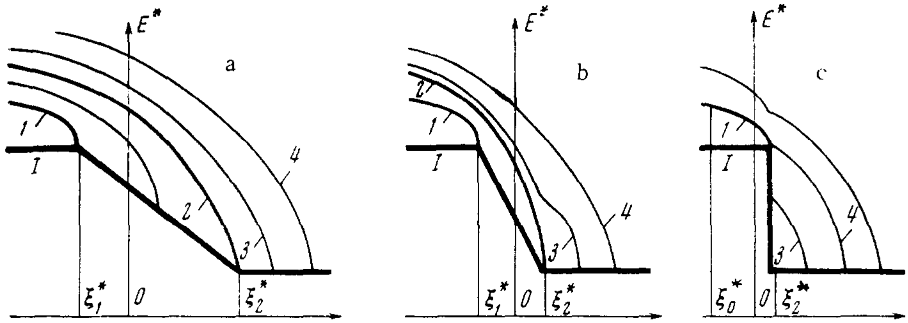

Fig. 3

2. We turn now to flows in which the current density $j < 0$ and the mobility coefficient b increases, as previously. Let $u =$ const. Integral curves related to this case are shown in Fig. 2. Only the part of this diagram lying above the isocline, where $b^*E^* + R_q u^* > 0$ and $dE^* / d\xi < 0$, have a physical meaning. We denote by *1* and *2* the integral curves which intersect the isocline at $\xi = \xi_1$ and $\xi = \xi_2$, respectively.

If a certain cross section $\xi = \xi_0 < \xi_1$ of the stream the electric field is defined by a curve lying between curve *2* and the isocline (above the latter), the field E^* decreases with increasing ξ up to the point of its intersection with the isocline. From thereon the curve runs in the direction of decreasing ξ. For the considered initial conditions defining the field structure a field with increasing b does not, generally, exist or may exist only along part of the segment $\xi_1\xi_2$.

If at a certain cross section $\xi = \xi_0$ the electric field is defined by an integral curve lying at a sufficient distance above the isocline $(b^*E^* + R_q u^* \gg \varepsilon)$, then the slope of that curve begins to decrease from point $\xi = \xi_1$. When the length of segment $\xi_1\xi_2$

tends to zero at constant b_1 and b_2, the related integral curves define the structure of a flow with a jump of the mobility coefficient. Such curves have no vertical tangents, hence the charge density q is finite and the surface charge intensity $\sigma = 0$. In this case the electric field is continuous, $E_1 = E_2$ (upstream of the discontinuity $b_1E_1 + u_1 \neq 0$). The charge density q is calculated by the second of Eqs. (1.5).

Let us assume that the electric field $E^* (\xi_3)$ at a certain section $\xi = \xi_3 > \xi_2$ is specified so as to satisfy the inequality

$$-\frac{R_q u^* (\xi_2)}{b^* (\xi_2)} < E^* (\xi_3) < -\frac{R_q u^* (\xi_1)}{b^* (\xi_1)} \tag{2.1}$$

We set in this and all subsequent formulas $u^* = 1$ for $u = \text{const}$. Let us consider the behavior of integral curves which may correspond to the selected field $E^* (\xi_3)$. These curves are denoted in Fig. 2 by *3*. The slope of integral curves along the stretch of decreasing ξ down to $\xi = \xi_2$ is small, since $|\varepsilon| \ll 1$. From the point $\xi = \xi_2$ the mobility coefficient begins to decrease, hence the term $b^*E^* + R_q u^* > 0$ also decreases, while the derivative $dE^* / d\xi$ increases. The integral curves approach the isocline and in its proximity turn abruptly to continue along its ε-neighborhood. They cannot intersect the isocline and move away from it. From point $\xi = \xi_1$ the mobility no longer decreases, field E^* continues to increase, the integral curve turns and follows the direction of the isocline, and the order of magnitude of the derivative $|dE^* / d\xi|$ becomes equal $|\varepsilon|$.

Let us examine to what kind of discontinuity corresponds the flow defined by these integral curves. When the length of segment $\xi_1 \xi_2$ and the width of the ε-neighborhood tend to zero, the integral curves approach the isocline, and at the limit with $\xi \leqslant \xi_2$ the integral curves *1* and *3* merge with the isocline. For this kind of discontinuity the field E_1 upstream of the discontinuity cannot be arbitrarily specified, there must exist there the relationship (obtained from the equation of the isocline)

$$u_1 + b_1 E_1 = 0 \tag{2.2}$$

The electric field downstream of such discontinuity may be arbitrarily selected, provided it satisfies inequalities (2.1) and is selected within the limits of these. The surface charge is calculated by formula

$$\sigma = \frac{E_2 + u_1 / b_1}{4\pi} \tag{2.3}$$

It is convenient to examine in this case the behavior of integral curves in the plane $E^* \xi^*$, where $\xi^* = x / L$, when $\varepsilon = 4\pi j L / b_1 E_1^2$. Here L is a certain characteristic length which remains constant when the length of segment $\xi_1^* \xi_2^* \to 0$. We assume that parameter $\varepsilon = \text{const}$ is not small. The integral curves lying above the isocline are shown in Fig. 3, a. With constant b_1 and b_2 and decreasing $|\xi_2^* - \xi_1^*|$ the integral curves are distorted, as shown in Fig. 3, b. The integral curve *2* presses along segment $\xi_1^* \xi_2^*$ against the isocline and for $\xi^* < \xi_1^*$ against integral curve *1*. Part of integral curves lying to the right of curve *2* (curve *3*) is also distorted when $\xi^* \leqslant \xi_2^*$. At the limit, when $|\xi_2^* - \xi_1^*| \to 0$, integral curves *1* and *2* as well as the curves lying between these two merge with the integral curve *1* and the related section of the isocline (Fig. 3, c). We shall denote the integral curve passing through point $\xi^* = \xi_2^*$, where $E^* = -R_q u^*(\xi_1^*) / b^* (\xi_1^*)$, by *4*. With decreasing ξ and $|\xi_2^* - \xi_1^*| \to 0$ the integral curves *3* lying to the left of curve *4* approach the vertical part of the isocline, merge with it, and follow it for $\xi^* = \xi_2^*$ and for $\xi^* < \xi_2^*$

run along the integral curve I (Fig. 3, c).

Let us consider the example of a flow with a mobility jump which corresponds to the described behavior of integral curves. Let at a certain section $\xi^* = \xi_0^* < \xi_1^*$ the electric field be defined by the integral curve I (Fig. 3, c). With increasing ξ^* the field E^* diminishes and the charge density q^* increases. At $\xi^* = \xi_2^*$ the mobility coefficient suffers a discontinuity and the integral curve penetrates the discontinuity with an infinite derivative ($|dE^* / d\xi^*| \to \infty$ when $\xi^* \to \xi_2^*$). This means that the charge density immediately ahead of the jump is infinitely high. Further on the integral curve follows the vertical part of the isocline and then can join any integral curve 3. By specifying the electric field E_2^* downstream of the discontinuity we determine the particular integral curve 3 which defines the variation of E^* in that region and also σ (Formula (2.3)).

If along segment ξ_1 ξ_2 the velocity $u = u(x)$ either decreases or increases, so that the isocline slope is negative all along that segment, all of the results derived in Sect. 2 remain unchanged. The considered flow defines the structure of the evaporation jump (b increases) in electrohydrodynamics with a small interaction parameter. If the velocity increases with increasing ξ at such rate that the isocline slope becomes positive, the pattern of integral curve behavior changes (see Sect. 4).

We note that discontinuities of the kind considered here, when the field upstream of the discontinuity must satisfy Eq. (2.2) and downstream of the wave is specified also, occur in electrohydrodynamic shock waves with constant mobility coefficients. In fact, if the velocity within the shock wave structure decreases (e.g. for small interaction parameters), while $j < 0$ and $E < 0$, the integral curves in the $E^*\xi$-plane behave in exactly the same manner, as in the case considered above.

Shock waves whose structure upstream of the wave front requires the specification of particular conditions are known in gasdynamics (condensation jumps) [5] and in magnetohydrodynamics [6].

3. Let us consider the case in which mobility diminishes. We assume that u = const and the current density $j > 0$. Line I (Fig. 4) represents the isocline along which the derivative $dE^* / d\xi$ becomes infinite. Only the part of Fig. 4 which lies below the isocline, where the integral curves are shown by solid lines, has any physical meaning. It is seen that with decreasing b^* the integral curves mildly slope toward the ξ-axis.

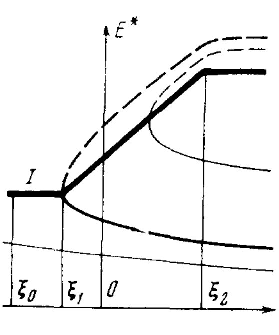

Fig. 4

When $j > 0$, the ions are carried by neutral particles by friction against the electric field force. With decreasing mobility the effect of friction increases and the electric field force decreases, the term ($u + bE$) increases, and the charge density diminishes and with it the derivative dE / dx, also, diminishes. In dimensionless coordinates the term $b^* E^* + R_q u^* < 0$, while increasing in absolute value with increasing ξ, and the derivative $|dE^* / d\xi|$ decreases. The above relates also to the case in which in the region of decreasing mobility the velocity u increases.

When the region ξ_1 ξ_2 is small, the related flow may be used as a model of the electrohydrodynamic flow which takes place in a condensation jump. Let us consider the flow of a mixture of a neutral gas with ions containing a supersaturated steam. We denote the mobility coefficient by b_1. The condensation of steam occurs in a certain narrow region, where the ions adhere to fluid droplets moving at a velocity close to that of gas and the mobility coefficient b markedly decreases. It can be seen that the relationships in an electrohydrodynamic condensation jump are the same as in an evaporation jump (1.7) ; the remainder $w_{01} - w_{02}$ is the quantity of heat released by condensation. As in the case of electrohydrodynamic snock waves [1] and of evaporation jumps considered in Sects. 1 and 2, the system of equations defining the discontinuity is not closed: the equation for determining the surface charge σ at the discontinuity front and, consequently, of the electric field normal component downstream of the discontinuity is missing. This equation can be obtained by analyzing the pattern of integral curves in the $E^*\xi$-plane with $u\ (x)$ as the variable.

It is known in gasdynamics [5] that in subsonic condensation jumps the normal velocity component of gas downstream of the jump front is greater than that upstream of the front

$$u_2 > u_1,\ \rho_2 < \rho_1,\ u_1 < a_1,\ u_2 < a_2,\ p_2 < p_1$$

In supersonic condensation jumps the normal component of gas velocity upstream of the front is greater than downstream of it

$$u_2 < u_1,\ \rho_2 > \rho_1,\ u_1 > a_1,\ u_2 > a_2,\ p_2 > p_1$$

Let us first consider the case of monotonically increasing velocity in the electrohydrodynamic condensation jump. When the interaction parameter is small, this corresponds in gasdynamics to a subsonic condensation jump. The pattern of integral curve behavior is the same as for $u = \text{const}$ (Fig. 4). The derivative $|dE^* / d\xi|$ decreases in region $\xi_1\xi_2$ with increasing ξ at a faster rate than in the case of $u = \text{const}$. When $l \to 0$, the region of variation of b narrows and the integral curves in Fig. 4 have nowhere vertical tangents. Hence for $j > 0$ the charge density is finite, its intensity $\sigma = 0$ throughout the subsonic condensation jump, and the electric field $E_1 = E_2$ is continuous. The charge density q_2 is determined by the second of Eqs. (1.5).

The integral curves in Fig. 4 also define the solution for the evaporation jump structure for $j > 0$. Since the mobility within a jump increases at a lower rate than the velocity, the slope of the isocline corresponds to that shown in Fig. 4 (see Sect. 1).

Let us consider the case of monotonically decreasing velocity in an electrohydrodynamic condensation jump. When the interaction parameter is small, this corresponds in gasdynamics to supersonic condensation jumps. Two different solutions are possible in this case.

Let the isocline defined by Eq. (1.3) behave as shown in Fig. 4, i.e. its slope in region $\xi_1\xi_2$ is positive. The behavior of integral curves in the $E^*\xi$-plane is of the same pattern as in the case in which the velocity in the jump increases (Fig. 4).

It was shown above that in a jump whose structure is defined by these integral curves the occurrence of a surface charge is impossible : $\sigma = 0$ and $E_1 = E_2$. If the isocline slope (1.3) is negative, it behaves in region $\xi_1\xi_2$ as shown in Fig. 1. Since in this case $j > 0$, only the branches of integral curves lying below the isocline have a physical meaning.

The pattern in this case is the same as that considered in Sect. 1 for an evaporation jump (when b increases and u decreases) and the behavior of integral curves is that shown in Fig. 1. With this pattern of integral curve behavior (see Sect. 1) the formation of surface charge at the condensation jump front is possible. The charge intensity is determined by the second formula of (1.6). To calculate the electric field downstream of the jump front we use the first formula of (1.6). The system of equations determining the condensation jump (1.7) is thus closed.

4. Let us now consider the case of decreasing mobility coefficient and $j < 0$. Let u = const. Only the curves lying above the isocline, where $b^*E^* + R_q u^* > 0$ have physical meaning. Let at a certain cross section $\xi = \xi_0 < \xi_1$ of the flow the electric field E^* be defined by an integral curve lying above the isocline between curves *1* and *2* which intersect the isocline at $\xi = \xi_1$ and $\xi = \xi_2$, respectively. The mobility coefficient b^* begins to decrease from point $\xi = \xi_1$, and the term $b^*E^* + R_q u^*$ tends to zero. The integral curves intersect the isocline between points ξ_1 and ξ_2 and then continue in the direction of decreasing ξ. In this case no flow structure exists in the region of decreasing mobility. The physical meaning of this is as follows. With decreasing mobility b the friction between ions and neutral particles increases so that the electric field is no longer capable of moving the ions. The velocity of the latter decreases and the charge density tends to infinity. Let at a certain cross section $\xi = \xi_0$ of the flow the electric field E^* be defined by a point lying above curve *2*. With decreasing b the slope of integral curves in the $E^*\xi$-plane and, consequently, also the charge density increase. However the intersection with the isocline occurs at $\xi > \xi_2$, and the flow structure along segment $\xi_1\xi_2$ exists. When the length of segment $\xi_1 \xi_2$ tends to zero, the integral curves have no vertical tangents, hence the surface charge density along the incipient jump is zero and the normal component of the electrical field is continuous, while the charge density q suffers a discontinuity.

Let $u^* = u^*(\xi)$ be a given function. We assume the isocline slope along segment $\xi_1\xi_2$ to be positive. Such flow corresponds either to a subsonic (u increases at the jump) or to a supersonic condensation jump (u decreases so that the tangent remains positive along segment $\xi_1\xi_2$). Then the described jumps are either nonexistent or they are of the gasdynamic kind : $\sigma = 0$ and $E_1 = E_2$, and q_2 is determined by the second of Eqs. (1.5). These integral curves also define the evaporation jump structure, in which $j < 0$, mobility increases, and the velocity increases at such high rate that the isocline slope along segment $\xi_1\xi_2$ remains positive.

Let the velocity in a supersonic condensation jump decrease at such rate that the isocline slope along segment $\xi_1\xi_2$ becomes negative, as shown in Fig. 2. Only the upper part of this figure, where $b^*E^* + R_q u^* > 0$. The flow in the jump is then the same as in an evaporation jump, when $j < 0$, the mobility coefficient b increases, and the velocity in the jump decreases. Condensation jumps are then possible ; their existence is only possible, if the electric field upstream of the jump satisfies the relationship $u_1 + bE_1 = 0$. The electric field downstream of such jump must be specified within limits defined by the inequalities (2.1) (see Sect. 2). Condensation jumps along which no surface charge develops, i.e. $\sigma = 0$ and $E_1 = E_2$ are also possible (the conventional gasdynamic condensation jump).

5. Let us consider the flow, when the electric field $E > 0$ (velocity $u > 0$). It follows from Eqs. (1.1) that in this case a surface charge cannot exist, the term $u + bE \neq 0$, the condensation and evaporation jumps are purely gasdynamic, and the charge density q_2 is subject to discontinuity, as shown by the second of Eqs. (1.5). However in regions of mobility variation a considerable increase of the charge may occur. Let in in region x_1x_2 the mobility decrease from b_1 to zero and $u = \text{const}$. At point $x = x_2$ the current density is $j = q_2u = q_1 (u + b_1E_1)$. If $b_1E_1 \gg u$ $(R_q \ll 1)$, then $q_2 / q_1 = R_q^{-1} \gg 1$. An abrupt increase of charge takes place in region x_1x_2. If $u = u(x)$, then for $R_q \ll 1$ we have the relationship

$$\frac{q_2}{q_1} = \frac{b_1E_1}{u_2} = \frac{1}{R_q}\frac{u_1}{u_2}$$

This formula is also valid for condensation jumps. If this takes place at decreasing velocity in the jump, the charge density increases even more than for $u = \text{const}$. If, on the other hand, the velocity in a condensation jump increases, the charge density may either increase or decrease. The variation of the electricity field in region x_1x_2 is proportional to the width of that region and can be considerable. If $x_2 - x_1 \to 0$, the field in condensation and evaporation jumps is, however, continuous.

If the mobility increases from a low value to a certain value b_2, so that $u_2 \ll b_2E_2$, the charge density decreases, hence

$$\frac{q_2}{q_1} = R_q\frac{b_1}{b_2} \qquad (b_1E_1 < b_2E_2)$$

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